B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering)

Term-End Examination

June, 2007

ET-101(A) : MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are **compulsory**.*

1. Answer any ***five*** of the following : *5×4=20*

(a) Evaluate the following limits :

(i) $\lim_{x \to 3} \left[\frac{2}{x^2 - 4x + 3} - \frac{1}{x - 3} \right]$

(ii) $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$

(b) If $y = \cos(m \sin^{-1} x)$, prove that $(1 - x^2) y_2 = x y_1 - m^2 y$. Using Leibniz's theorem find y_{n+2}.

(c) If the tangent to the curve

$$\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$$

cuts off intercepts p and q on the axes of x and y respectively, show that $\frac{p}{a} + \frac{q}{b} = 1$.

(d) Find the points of continuity and discontinuity of the function, f, given by

$$f(x) = \begin{cases} \frac{|x-3|}{x^2-9}, & \text{when } x \neq 3 \\ \frac{1}{6}, & \text{when } x = 3 \end{cases}$$

(e) Find the maximum and minimum values of the function, f, given by

$$f(x) = \frac{4}{x} - \frac{1}{x-1} \quad \forall x \in \mathbf{R} - \{0, 1\}.$$

(f) if $u = f(ax^2 + 2hxy + by^2)$ and $v = g(ax^2 + 2hxy + by^2)$, prove that

$$\frac{\partial}{\partial y}\left(u \frac{\partial v}{\partial x}\right) = \frac{\partial}{\partial x}\left(u \frac{\partial v}{\partial y}\right).$$

2. Answer any ***four*** of the following : *4×4=16*

(a) Evaluate the following integrals :

(i) $\int \frac{x + \sin x}{1 + \cos x}\, dx$

(ii) $\int (x^3 \cot^{-1} x)\, dx$

(b) (i) Evaluate $\int_0^1 \frac{x \cos^{-1} x}{\sqrt{1-x^2}}\, dx$.

(ii) Evaluate $\int_1^3 2x^3\, dx$ using integral as a sum.

(c) Find the area included between one arch of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ and its base.

(d) Taking four subdivisions of the interval [0, 4], find an approximate value of $\int_0^4 \frac{x}{1+x^2}\,dx$, using the Trapezoidal Rule.

(e) Is the differential equation
$2x\cos^2 y\,dx + (2y - x^2 \sin 2y)\,dy = 0$ exact ? If yes, solve it.

3. Answer any ***four*** of the following : *4×4=16*

(a) A force $\vec{F} = 4\hat{i} - \hat{j} + 3\hat{k}$ is applied at the point P (3, 1, 2). The moment of this force about the point Q (t, 1, 2t) is $6\hat{i} + 9\hat{j} - 5\hat{k}$. Find t.

(b) If $\vec{F} = x^2 y\hat{i} + xz\hat{j} + 2yz\hat{k}$; find div curl $\vec{F}$.

(c) Determine the constants a, b and c so that the vector field given by

$$\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$$

is conservative.

(d) Find the directional derivative of the function, $\phi(x, y, z) = xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t$, $y = t^2$, $z = t^3$ at the point (1, 1, 1).

(e) Evaluate $\int \vec{F} \cdot d\vec{r}$ along the curve,

$x^2 + y^2 = 1,\ z = 1$ in the positive direction from $(0, 1, 1)$ to $(1, 0, 1)$, where

$\vec{F} = (2x + yz)\hat{i} + xz\hat{j} + (xy + 2z)\hat{k}$.

(f) For the function f defined by

$$f(x, y) = \begin{cases} \dfrac{1}{y^2}, & \text{when } 0 < x < y < 1 \\ -\dfrac{1}{x^2}, & \text{when } 0 < y < x < 1 \end{cases}$$

Show that $\int_0^1 dx \int_0^1 f\,dy \neq \int_0^1 dy \int_0^1 f\,dx$.

4. Answer any ***three*** of the following : *3×6=18*

(a) Apply the Cayley – Hamilton theorem to decide whether the following matrix is invertible. If it is, obtain its inverse. Otherwise, obtain its rank.

$$\begin{bmatrix} 2 & 2 & -6 \\ 2 & -1 & -3 \\ 2 & -1 & 1 \end{bmatrix}$$

(b) Is the set

$S = \{(1, 1, -2, 1), (3, 0, 4, -1), (-1, 2, 5, 2)\}$

linearly independent in $\mathbf{R}^4$? Check further whether $(4, -5, 9, -7)$ belongs to L(S) or not.

(c) Express the matrix $A = \begin{bmatrix} 2 & 2 \\ 4 & 0 \end{bmatrix}$ in a canonical diagonal form.

(d) Check whether the following transformations are linear. Further for those that are, check the rank-nullity theorem.

(i) $T : \mathbf{R} \to \mathbf{R}^3 : T(x) = (x, x^2, x^3)$

(ii) $T : \mathbf{R}^2 \to \mathbf{R}^3 : T(x_1, x_2) = (x_1, x_1 + x_2, x_2)$